Direct Unfalsified Controller Design - Solution Via Convex Optimization

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Abstract

This paper presents an algorithm for designing optimal unfalsified discrete linear time invariant (LTI) output feedback controllers directly from measured data. The approach uses a performance specification that corresponds to minimizing the time domain error between the desired and unfalsified closed loop transfer functions. No assumptions about the plant are explicitly required - the only information needed is a plant input-output time history of length n. In particular, no assumptions are made about the linearity of the system. The identified controller is unfalsified with respect to the performance specification by all observed data. With some minor assumptions, the design problem can be written as a linear program (LP) that can be solved very efficiently to find the global optimum. The approach is demonstrated with a laboratory experiment.

1 Introduction

There has been much recent work in system identification using the concepts of *model unfalsification* [1]–[6]. These techniques focus on forming plant uncertainty models that are not provably inconsistent with observations, that is, the uncertainty models are unfalsified by the data. Controllers are then designed that meet specifications with respect to the unfalsified uncertainty models.

Modeling techniques that generate a model set consisting of a nominal plant model and uncertainty bounds on the nominal model [3] are of particular interest because they produce plant models that are compatible with robust control design techniques. Reference [7] shows that control designs based on these models can be conservative, resulting in low performance controllers. This conservatism arises because the unfalsified model set is a superset of the true plant model, and robust control techniques often meet specifications for a superset of the unfalsified model set. Unfortunately, each of these overbounds can be quite large.

The notion of unfalsification can also be applied directly to candidate controllers [4]. Without performing an additional experiment, it is possible to determine whether a given controller could not have met a certain performance specification if it had been connected to the system during the previous experiment. If the controller can not be eliminated (falsified) by the measured data, the controller is said to be unfalsified with respect to that data.

The use of an unfalsified model or an unfalsified controller in a control experiment does not guarantee that the closed-loop system will meet the desired performance specifications [1]. For example, the available data may not fully excite the plant that is to be controlled. When a model (or controller) is unfalsified, it means that no evidence is available that contradicts the model (or controller). This suggests that unfalsification techniques might perform well as part of an adaptive control scheme.

The application of the unfalsified control design concept within an adaptive control scheme has been proposed by many researchers [2], [6], [8]. Typically, the designer first proposes a large finite set of candidate controllers. One controller is implemented, and data is collected with the controller in the loop. This data is used to eliminate some of the candidate controllers by falsifying them with respect to a performance specification. If the operating controller is falsified by the data, a new unfalsified controller is implemented. The process is repeated until no further improvements are desired.

This paper builds on those results by formulating a convex programming problem using the set of fixed order linear time invariant output feedback controllers as the candidate set. The controller that achieves the best performance is chosen from the set of unfalsified controllers. The algorithm is demonstrated experimentally in a single iteration of data collection/controller design.

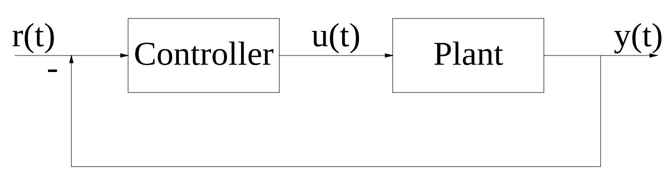

Figure 1: Standard Block Diagram

2 Background

In a recent work, Tsao and Safonov laid a theoretical framework for determining the subset of candidate controllers that are unfalsified with respect to observed data and a performance specification [5]. The main result of Tsao and Safonov's paper is quoted below, and serves as a starting point for this paper's contribution.

Unfalsified Control Problem: Consider input-output data collected from an unknown plant. The data could be collected "open-loop" or collected "closed-loop" with $u_{data}(t)$ generated by some operating controller. Let

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$y_{data}(t)$ represent the output corresponding to $u_{data}(t)$. Let $\mathcal{Y}$ and $\mathcal{U}$ represent functional spaces. Define the measurement data set

$$M_{data} \stackrel{\triangle}{=} \{(y,u) \in \mathcal{Y} \times \mathcal{U} \mid (y_{data}, u_{data}) \in (y,u)\} \quad (1)$$

that is, measurements of the plant input-output relationship are contained in a relation in $\mathcal{Y} \times \mathcal{U}$. The distinction between (y_{data}, u_{data}) and (y,u) is useful if one wishes to model imperfect measurements. For example, one possible choice of M_{data} could be

$$M_{data} = \left\{ (y,u) \in l_2 \times l_2 \;\middle|\; \begin{bmatrix} \|y - y_{data}\|_{rms} \\ \|u - u_{data}\|_{rms} \end{bmatrix} \leq \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \right\}$$

that is the true plant input-output data could be corrupted by noise with RMS norms β_2 and β_1 respectively.

Now consider the additional fictitious signal $r \in \mathcal{R}$ as shown in Fig. 1. Define the measurement information set

$$P_{data} \stackrel{\triangle}{=} \{(r,y,u) \in \mathcal{R} \times \mathcal{Y} \times \mathcal{U} \mid (y,u) \in M_{data}\} \quad (2)$$

Let the class of admissible control laws be

$$K_{admiss} \subset \mathcal{R} \times \mathcal{Y} \times \mathcal{U} \quad (3)$$

and define a performance specification set

$$T_{spec} \subset \mathcal{R} \times \mathcal{Y} \times \mathcal{U} \quad (4)$$

Then given P_{data}, K_{admiss}, and T_{spec}, find the set of controllers K such that $K \subset K_{admiss}$ is unfalsified with respect to P_{data} and T_{spec}.

Theorem 1 [5]: K is unfalsified with respect to P_{data} and T_{spec} iff

$$\forall \;\; (r_0, y_0, u_0) \in P_{data} \cap K, \;\; \exists \;\; (y_1, u_1) \;\; s.t.$$

$$(r_0, y_1, u_1) \in P_{data} \cap K \cap T_{spec}$$

Note that $(y_1, u_1) = (y_0, u_0)$ is allowed.
Proof: See [5].

As pointed out in [5] there are several advantages to this data-based control design approach:

1. The approach is nonconservative; *ie.* it gives “if and only if” conditions on the candidate controller set to be unfalsified.
2. The unfalsified set of candidate controllers is determined from past data only – *no candidate controller is implemented to determine if it is falsified.*
3. The test for controller unfalsification is “plant-model free.” No plant model is needed to test these conditions. The test depends only on the data, the controller and the specification.
4. The data that falsifies a controller may be open loop data or data generated by some other control law which may or may not be in the parametric set.

3 Problem formulation

Throughout this paper, it is assumed that all signals are scalar, finite duration, and are collected at uniform sampling instants, and

$$r \in \mathcal{R} \equiv l_2[0, n-1] \;, \quad y \in \mathcal{Y} \equiv l_2[0, n-1]$$

$$u \in \mathcal{U} \equiv l_2[0, n-1]$$

Thus r, y, and u can be represented by vectors in $\mathbb{R}^n$.

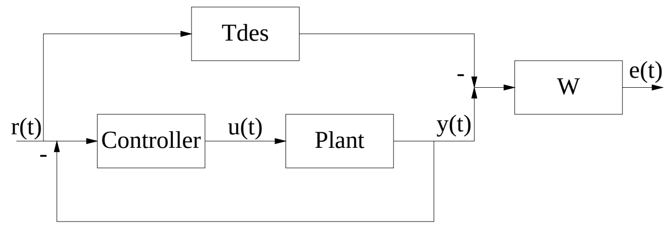

Figure 2: Block Diagram Illustrating T_{spec}

Let $y_{data} \in \mathbb{R}^n$ and $u_{data} \in \mathbb{R}^n$. Choose

$$P_{data} = \left\{ (r,y,u) \;\middle|\; \begin{bmatrix} y - y_{data} \\ u - u_{data} \end{bmatrix} = 0 \right\} \quad (5)$$

ie. there is no explicit noise model incorporated in P_{data}. The set of admissible control laws is chosen to be of the form

$$K_{admiss} = \left\{ (r,y,u) \;\middle|\; u = \frac{N(q^{-1})}{D(q^{-1})}(r - y) \right\} \quad (6)$$

where q is the forward shift operator, and $N(q^{-1})$ and $D(q^{-1})$ have arbitrary initial conditions. We further restrict $N(q^{-1})$ and $D(q^{-1})$ to have the same order. Choose

$$T_{spec} = \left\{ (r,y,u) \;\middle|\; \|W(q^{-1})(y - T_{des}(q)r)\|_\infty \leq \alpha \right\} \quad (7)$$

Where $T_{des}(q)$ is the desired LTI closed loop transfer function of order k and $W(q^{-1})$ is an LTI weighting function. Both $T_{des}(q)$ and $W(q^{-1})$ have arbitrary initial conditions. Note that the signal infinity norm is employed, not to be confused with the H_∞ system norm. This cost function can be interpreted as $\|e\|_\infty \leq \alpha$, with e as shown in Fig. 2.

When choosing P_{data}, K_{admiss}, and T_{spec}, it is important to ensure that $\forall \; 0 \leq i \leq j \leq n-1$

$$\begin{aligned} (r,y,u) &\in P_{data} \Longrightarrow (r_{[i,j]}, y_{[i,j]}, u_{[i,j]}) \in P_{data} \\ (r,y,u) &\in K_{admiss} \Longrightarrow (r_{[i,j]}, y_{[i,j]}, u_{[i,j]}) \in K_{admiss} \\ (r,y,u) &\in T_{spec} \Longrightarrow (r_{[i,j]}, y_{[i,j]}, u_{[i,j]}) \in T_{spec} \end{aligned} \quad (8)$$

The above relations ensure that if a data set could not falsify a controller, then any subinterval of the data could not falsify the controller.

The choice of the signal infinity norm is significant in that it has this property. For example, if the RMS norm had been chosen, it would be required to consider all possible time truncations of the signal in the performance specification, that is

$$T_{spec} = \{(r, y, u) \mid \|e_{[i,j]}\|_{rms} \leq \alpha \ \forall\ 0 \leq i \leq j \leq n-1\}$$

which greatly increases the problem size of determining if $(r, y, u) \in T_{spec}$.

Applying Theorem 1, the problem of finding the "best" (smallest α) LTI output feedback controller of order m, relative degree zero, that unfalsifies the measured data is

$$\min_{(r,y,u),\alpha} \quad \alpha \tag{9}$$

subject to

$$(r, y, u) \in P_{data} \cap K \cap T_{spec} \tag{10}$$

$$K \subset K_{admiss} \tag{11}$$

4 Optimal controller as an LP

In order to simplify discussion, the SISO case will be treated, however the results can easily be extended to MIMO systems.

Theorem 2: The best (smallest α) unfalsified LTI output feedback controller of order m that satisfies T_{spec}, with $W(q^{-1}) = N(q^{-1})$, is found by solving the following linear program (LP).

$$\min_{r_{ic}, \tilde{y}_{\mathrm{ic}}, \tilde{u}_{\mathrm{ic}}, b, a, \alpha} \alpha \tag{12}$$

subject to

$$\begin{bmatrix} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 \\ -A_1 & -A_2 & -A_3 & -A_4 & -A_5 & A_6 \end{bmatrix} \begin{bmatrix} r_{ic} \\ \tilde{y}_{\mathrm{ic}} \\ \tilde{u}_{\mathrm{ic}} \\ b \\ a \\ \alpha \end{bmatrix} \leq \begin{bmatrix} B \\ -B \end{bmatrix}$$

where

$$A_1 = \begin{bmatrix} 0 & I \end{bmatrix} \tilde{\mathcal{T}}(t_{des}) \tag{13}$$

$$A_2 = \tilde{\mathcal{T}}\left(t_{des} - \begin{bmatrix} 1 & \mathbf{0}^T \end{bmatrix}^T\right) \tag{14}$$

$$A_3 = \tilde{\mathcal{T}}(t_{des}), \quad A_4 = (\mathcal{T}(t_{des}) - I)\tilde{\mathcal{T}}(y) \tag{15}$$

$$A_5 = \mathcal{T}(t_{des})\tilde{\mathcal{T}}\left(\begin{bmatrix} 0 & u_{[0,n-2]}^T \end{bmatrix}^T\right) \tag{16}$$

$$A_6 = -\mathbf{1}, \quad B = -\mathcal{T}(t_{des})\tilde{\mathcal{T}}(u)\begin{bmatrix} 1 \\ \mathbf{0} \end{bmatrix} \tag{17}$$

where $\mathbf{1} \triangleq [1\,1 \cdots 1]^T$, $\mathbf{0} \triangleq [0\,0 \cdots 0]^T$, t_{des} is the impulse response of $T_{des}(q)$, $\mathcal{T}(x)$ is the square lower triangular Toeplitz matrix of appropriate dimension

$$\mathcal{T}(x) \triangleq \begin{bmatrix} x(0) & 0 & \cdots & 0 \\ x(1) & x(0) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ x(n-1) & x(n-2) & \cdots & x(0) \end{bmatrix} \tag{18}$$

and

$$\tilde{\mathcal{T}}(x) \triangleq \mathcal{T}(x)\begin{bmatrix} I \\ \mathbf{0} \end{bmatrix} \tag{19}$$

The resulting controller is

$$C(q) = \frac{b_0 + b_1 q^{-1} + \cdots + b_m q^{-m}}{1 + a_1 q^{-1} + \cdots + a_m q^{-m}} \tag{20}$$

Furthermore, the LP is always feasible.

Proof: The optimization constraint of Eq. (10) implies

$$(r, y, u) \in P_{data} \tag{21}$$

thus

$$y = y_{data}, \quad u = u_{data} \tag{22}$$

Eqs. (10) and (11) imply

$$(r, y, u) \in K \subset K_{admiss} \tag{23}$$

thus Eq. (6) implies

$$N(q^{-1})r = D(q^{-1})u + N(q^{-1})y \tag{24}$$

Substituting Eqs. (22) and (24) into T_{spec}, and choosing $W(q^{-1}) \triangleq N(q^{-1})$ produces

$$(r, y, u) \in P_{data} \cap K \cap T_{spec} \iff$$
$$\|N(q^{-1})y - N(q^{-1})T_{des}(q)r\|_\infty \leq \alpha \iff$$

$$\|N(q^{-1})y - T_{des}(q)(D(q^{-1})u + N(q^{-1})y)\|_\infty \leq \alpha \iff \tag{25}$$

$$\|N(q^{-1})(1 - T_{des}(q))y - D(q^{-1})T_{des}(q)u\|_\infty \leq \alpha \tag{26}$$

It is interesting to note that the form of Eq. (26) is similar to that of an ARX identification problem with data $Z = [T_{des}(q)u, (1 - T_{des}(q))y]$ [9].

Consider an arbitrary relation such as $y(t) = G(q)u(t)$. For $y \in \mathbb{R}^n$, $u \in \mathbb{R}^n$, and ignoring initial conditions, this relation can be vectorized as

$$y = \mathcal{T}(g)u \tag{27}$$

where g is the impulse response of $G(q)$. If $G(q)$ is order p, then arbitrary initial conditions on $G(q)$ can be reflected in y by incorporating arbitrary $u_{ic} \in \mathbb{R}^p$

$$y = \begin{bmatrix} 0 & I \end{bmatrix} \mathcal{T}(g) \begin{bmatrix} u_{ic} \\ u \end{bmatrix} \tag{28}$$

Observing that $N(q^{-1})$ and $D(q^{-1})$ are order m, and $T_{des}(q)$ is order k, then Eq. (25) can be written incorporating initial conditions in vector form as

$$-\alpha\mathbf{1} \leq [0\ I]\,\mathcal{T}\left(\begin{bmatrix} b \\ \mathbf{0} \end{bmatrix}\right)\begin{bmatrix} y_{ic} \\ y \end{bmatrix} - [0\ I]\,\mathcal{T}(t_{des})$$
$$\begin{bmatrix} r_{ic} \\ [0\ I]\,\mathcal{T}\left(\begin{bmatrix} 1 \\ a \\ \mathbf{0} \end{bmatrix}\right)\begin{bmatrix} u_{ic} \\ u \end{bmatrix} + [0\ I]\,\mathcal{T}\left(\begin{bmatrix} b \\ \mathbf{0} \end{bmatrix}\right)\begin{bmatrix} y_{ic} \\ y \end{bmatrix} \end{bmatrix}$$
$$\leq \alpha\mathbf{1} \tag{29}$$

where b represents the $m+1$ coefficients of $N(q^{-1})$, a represents the m coefficients of $D(q^{-1})$, t_{des} is the impulse response of $T_{des}(q)$, y_{ic} and u_{ic} are arbitrary sequences of length m representing the initial conditions of $N(q^{-1})$ and $D(q^{-1})$, and r_{ic} is an arbitrary sequence of length k, representing the initial conditions of $T_{des}(q)$.

Note that for arbitrary $a \in \mathbb{R}^n$, $b \in \mathbb{R}^n$, $c \in \mathbb{R}^n$

$$a = \mathcal{T}(b)c \iff a = b \star c \iff a = c \star b \iff a = \mathcal{T}(c)b$$

Also note that if $b_j = 0 \ \forall \ j > p$, that is, the operator T(b) is FIR, and $b_0 \neq 0$ then

$$a = \begin{bmatrix} 0 & I \end{bmatrix} \mathcal{T}(b) \begin{bmatrix} c_{\mathrm{ic}} \\ c \end{bmatrix} = \mathcal{T}(b)c + \begin{bmatrix} \tilde{c}_{\mathrm{ic}} \\ \mathbf{0} \end{bmatrix}$$

for some $\tilde{c}_{\mathrm{ic}} \in \mathbb{R}^p$ which is a modified form of c_{ic}. Incorporating these ideas, Eq. (29) can be rewritten as

$$\begin{aligned} -\alpha \mathbf{1} \leq \tilde{\mathcal{T}}(y)\, b + \begin{bmatrix} \tilde{y}_{\mathrm{ic}} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} 0 & I \end{bmatrix} \tilde{\mathcal{T}}(t_{des})\, r_{ic} \\ -\mathcal{T}(t_{des}) \tilde{\mathcal{T}}(u) \begin{bmatrix} 1 \\ a \end{bmatrix} - \tilde{\mathcal{T}}(t_{des})\, \tilde{u}_{\mathrm{ic}} \\ -\mathcal{T}(t_{des}) \tilde{\mathcal{T}}(y)\, b - \tilde{\mathcal{T}}(t_{des})\, \tilde{y}_{\mathrm{ic}} \leq \alpha \mathbf{1} \end{aligned} \quad (30)$$

Eq. (30) describes the set of controllers that is unfalsified by the data. The “best” controller in the unfalsified set is the one that produces the smallest α. Optimizing Eq. (30) with respect to the variables given in (12) results in the LP given in Theorem 2.

For many practical choices of $T_{des}(q)$, A_2 and A_3 will have nearly linearly dependent columns, resulting in numerical problems for some LP solvers. A simple solution is to eliminate $\tilde{u}_{\mathrm{ic}}$ from the LP, which, experience indicates, does not have a significant impact on the optimal controller.

5 Example: Flexible structure control

A commercially available version of the classic “three disk” torsional plant is used to demonstrate the direct unfalsified control technique presented in this paper. An Educational Control Products Model 205 is used. It consists of three rotational masses connected by two torsional springs. Three encoders measure the angular rotation of each of the disks. The system has one actuator which applies a torque to the bottom disk. Fig. 3 shows a schematic of the system.

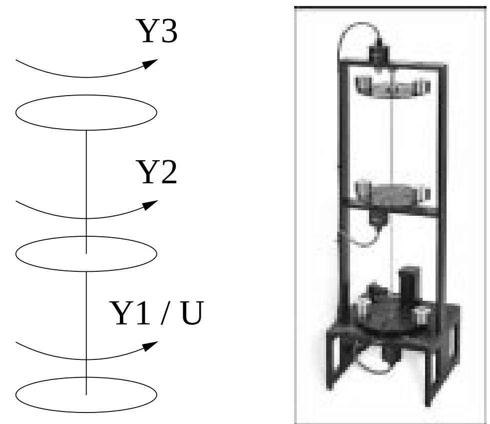

Figure 3: Schematic and photograph of the testbed.

The following investigates the SISO collocated tracking problem. The objective is to design a controller that forces y_1 to track a reference r. The sample rate was chosen to be 64 Hz, well above the highest frequency dynamics in the system.

Fig. 4 shows open-loop data that was collected from the experimental hardware, with

$$u(t) = A(1 + \sin(\omega_1 t + \theta_1) + \cdots + \sin(\omega_{32} t + \theta_{32}))$$

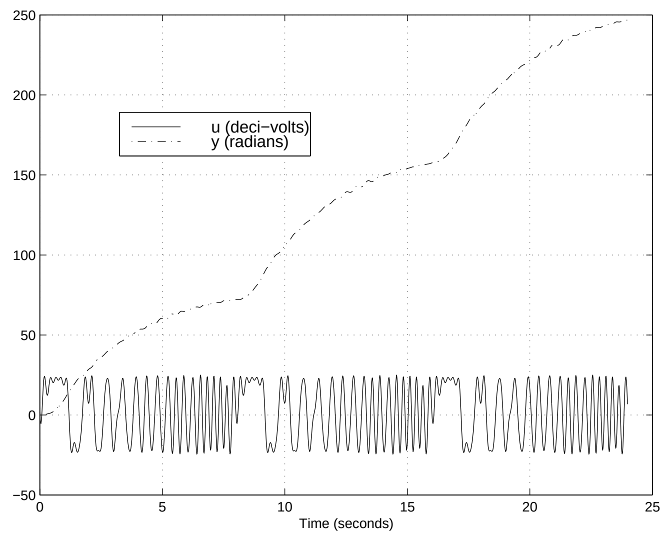

Figure 4: Measured (y, u)

where $\omega_k = k\pi/4$, and θ_k is optimized in order to minimize $\|u(t)\|_\infty$. This single run of data was the only information used to design the unfalsified controller. T_{des} was chosen in continuous time

$$T_{des}(s) = \frac{(2\pi)^2}{s^2 + 4\pi s + (2\pi)^2} \quad (31)$$

that is, a critically damped second-order plant with $\omega_n = 2\pi$ rad/sec (1 Hz). $T_{des}(q)$ is formed by a zero-order-hold conversion of $T_{des}(s)$. $N(q^{-1})$ and $D(q^{-1})$ were chosen to be third-order ($m = 3$).

The LP in Theorem 2 was solved resulting in $\alpha = 0.0036$, which indicates that a controller has been designed which unfalsifies the observations with respect to T_{des} with very small error (small $\|e\|_\infty$). The resulting controller is

$$C(q) = \frac{0.7824 - 2.289q^{-1} + 2.269q^{-2} - 0.763q^{-3}}{1 - 2.803q^{-1} + 2.649q^{-2} - 0.8433q^{-3}} \quad (32)$$

The controller is able to track a π radian step command reasonably well, as shown in Fig. 5.

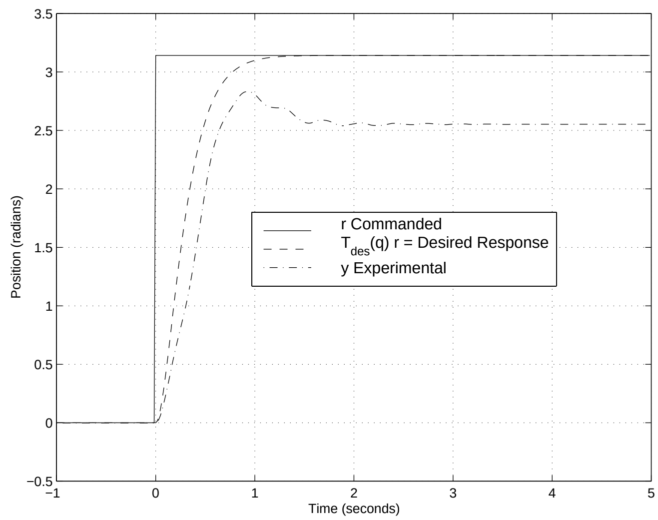

Figure 5: Experimental Step Response, Direct Unfalsified Control

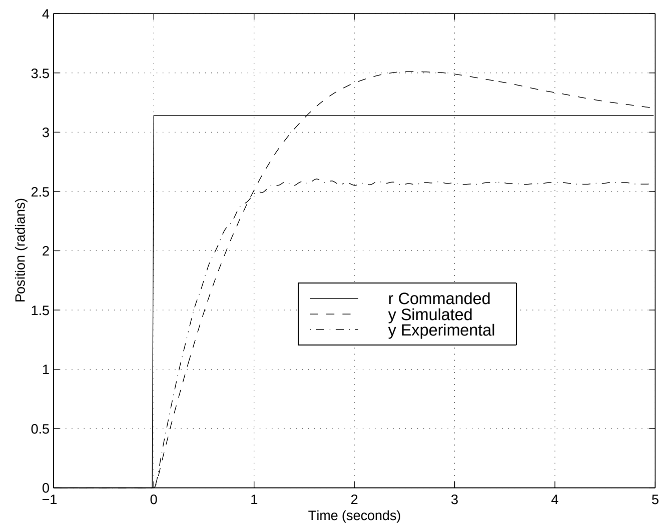

Figure 6: Experimental Step Response, LQG Control

In order to highlight the potential advantages of direct unfalsified control, an experimental comparison using a model based control design was performed. Simple physical modeling and extensive parameter identification produced a sixth-order LTI plant model. An LQG controller was designed using this plant model, resulting in the step response of Fig. 6.

The steady state error in both Fig. 5 and Fig. 6 is due to a stiction nonlinearity in the plant, and is typical of all linear controllers implemented on this plant. Although there are small differences in the rise time and overshoot of the two designs, similar performance is achieved with low order DUC ($m = 3$) as is achieved with "full order" LQG ($m = 6$). Furthermore, DUC required no knowledge of the plant.

Finally, if the plant model is concatenated with the DUC controller of Eq. (32), the resulting closed loop plant is predicted to be unstable. This is clearly not the case, as demonstrated in Fig. 5. A model based control design technique could never have produced the DUC controller.

6 Conclusions

It was shown that the problem of designing an optimal discrete linear time invariant output feedback controller directly from data can be cast as an LP. No assumptions about the plant are required - the only information needed is a plant input-output time history of length n. A key assumption is that the performance specification is weighted by $W(q^{-1}) = N(q^{-1})$. The identified controller is unfalsified with respect to the performance specification by all observed data.

Experimental results emphasize several main points regarding direct unfalsified control.

- The direct unfalsified control is able to produce controllers that have reasonable performance when compared to traditional design techniques.
- The direct unfalsified control makes no assumptions about the plant's linearity, thus the DUC controller can take into account nonlinear effects (such as stiction) when choosing the best controller.
- DUC can choose a controller that performs well experimentally, but would have been discarded as "unstable" by more traditional adaptive control techniques.

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